SNO-STR-91-025

Extraction of CC NC and ES Signals - III
Multi-Year
Three Parameter Maximum Likelihood Analysis

W. Frati and E. Beier
Univ of Pennsylvania

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1 Introduction

The study presented in SNO-STR-91-19, which analyzed year 2 and year 3 separately, is extended to combining all three years of running into one analysis. This in effect places constraints on the elastic scattering and charged current amplitudes in year 3 to those values obtained from year 1 and year 2, and reduces the strong negative correlation between the CC and NC (D_2O + Chlorine) extracted amplitudes when analyzing year 3 alone.

The analyses are performed for an acrylic radioactivity that is X1, X10 and X100 the white book value and only events with energy greater than 6 MeV and reconstructed radius less the 7 meters are used. An analysis with a 5 MeV threshold was presented in SNO-STR-91-19, with the caveat that the low energy beta-gamma events are excluded from the analysis, and is not repeated in this report.

2 Generalized Maximum Likelihood for Analyzing Three Years Simultaneously

There are now seven amplitudes to be extracted

w_1: Elastic Scattering in the H_2O volume (outside the acrylic)

w_2: Elastic Scattering in the D_2O volume (inside the acrylic)

w_3: Charged Current

w_4: Neutral Current D_2O only (year 2)

w_5: Acrylic Background D_2O

w_6: Neutral Current D_2O + Chlorine (year 3)

w_7: Acrylic Background D_2O + Chlorine

Note that the neutral current within the acrylic volume is the sum of the solar signal and that produced by the internal radioactivity, and this analysis is completely insensitive to the difference.

The signal function $G(N, u, R^3)$ is now given by

$\sum_{k=1}^{2} w_k F_k(N, u, R^3)$ for events in year 1; or

$\sum_{k=1}^{5} w_k F_k(N, u, R^3)$ for events in year 2; or

$\sum_{k=1,2,3,6,7} w_k F_k(N, u, R^3)$ for events in year 3

and the log of the likelihood function by

$$L(w_k) = \left[\sum_{n=1}^{Nev} lnG^{yr1} - \sum_{k=1}^{2} w_k\right] + \left[\sum_{n=1}^{Nev} lnG^{yr2} - \sum_{k=1}^{5} w_k\right] + \left[\sum_{n=1}^{Nev} lnG^{yr3} - \sum_{k=1,2,3,6,7} w_k\right]$$

Note that this analysis assumes that there is no variation in the solar signal from year to year, e.g. the amplitude for the elastic scattering in the H_2O volume outside the acrylic is w_1 for each of the three years, even though different events are used for each of the three years. Similarly the same weighting w_3 is used for the CC events in year 2 and year 3, although year 2 and year 3 have different event sets.

3 X1 ACRYLIC RESULTS

Table I summarizes the results when the three years are analyzed individually. The INPUT column gives the number of "data" events generated, and the remaining columns show the number of events extracted by the Maximum Likelihood Analysis (MLA), with the statistical errors given in parenthesis. Table II summarizes the results when the analysis is performed for three year analysis and all combinations of a two year analysis. Included in Tables I and II are a) the correlation coefficient of the error matrix between the neutral current(D_2O + Cl) and charged current and b) the extracted neutral current/charged current ratio with its statistical error in parenthesis. The correlation coefficient is included in the calculation of the statistical error of the NC/CC ratio.

3.1 Year 1 Only

The statistical errors for the elastic scattering in the H_2O and "D_2O " volumes signals are fairly close to what one would get by taking the square root of the number of events, showing that the correlation between the two is small. The statistical error is 9.9% (7.9%) for the elastic scattering signal in the H_2O (D_2O)

3.2 Year 2 Only

1) The statistical accuracy of the elastic scattering signal in the H_2O degrades marginally from year 1 to 10.6%.

2) The statistical accuracy of the elastic scattering signal in the D_2O goes from 7.9% in year 1 to 11.4%. This is due to the fact that the CC and NC signals occur in the same volume and are "backgrounds" to the ES signal.

3) The charged current amplitude is determined with a 2.6% statistical accuracy.

4) A rough determination of the neutral current amplitude is obtained with a statistical accuracy of 23.7%.

5) No measurement on the acrylic radioactivity is possible, i.e. the statistical error in the extracted amplitude exceeds the amplitude.

3.3 Year 3 Only

1) The statistical accuracy of the H_2O and D_2O elastic scattering signals degrades a bit over year 2.

2) The statistical accuracy of the charged current is a a factor of two poorer than year 2, going from 2.6 to 5.2%. This is a manifestation of the NC signal acting as a "background" to the CC signal, which occurs in the same volume of the detector.

3) The neutral current is determined with a 5.0% accuracy.

4) A 21.2 % statistical accuracy for the acrylic background is obtained.

5) The correlation coefficient of the error matrix between the neutral current and charged current is -.717. Including this in the determination of the error for the NC(D_2O + Cl)/CC ratio yield a 10.5% statistical error.

3.4 Multiple Year Analysis

Analyzing the three years together reduces the NC-CC correlation coefficient from −.717 to −.423, and the statistical accuracy of the NC/CC ratio is 5.6%, almost a factor of two improvement over the results obtained from analyzing year 3 alone. This is the effect of year 2 imposing a constraint on the CC signal signal. A similar result is obtained when year 2 and year 3 are combined, a result to be expected since year 1 does not affect the statistical accuracy of the NC and CC determinations.

4 X10 Acrylic

The results are listed in Tables III and IV and the conclusions are pretty much the same as for the X1 acrylic analysis. The only possible difference from the X1 acrylic case is in the NC/CC ratio which is 1.5 standard deviations off when analyzing year 3 alone, suggesting a systematic effect but the percentage error on the ratio (10.0%) is almost the same as for the X1 acrylic case.

Of course the statistical accuracy in determining the acrylic background has improved, and it would be possible to observe this at the four sigma level with year 2 data alone.

5 X100 Acrylic

Table V lists the results of the year 2, year 3 and year $1+2+3$ analyses. The year $1+2+3$ results for the X1 and X10 acrylic are included in Table V for easy comparison. The results for the various analyses are presented below.

5.1 Year 2 Only

The year 2 analysis is not significantly affected by the level of the acrylic radioactivity. The statistical errors for the X100 acyrlic case are somewhat higher than for the X1 case, but the extracted amplitudes all agree with those generated.

Fig 1a shows the radial (R^3) distributions of each of the component signals and Fig 1b shows the radial distribution of the "data" (coarse histogram and best fit (fine histogram) using the weightings extracted from the year $1+2+3$ analysis. Figs 2a and 2b (3a and 3b) are similar plots for the NPMT ($\cos(\theta_{sun})$) distributions. The b figures also show the generating function which often overlap the best fit.

The CC is still the dominant signal, but the acrylic background dominates the NC for R> 4 meters. In view of the fact that the NC and acrylic background have similar NPMT distributions and both are isotropic, it is somewhat surprising that the statistical accuracy of the extracted NC is 27.9% when compared to the X1 acrylic accuracy of 23.7%.

5.2 Year 3 Only

Figs 4a-4b, 5a-5b and 6a-6b are the R^3 NPMT and $\cos(\theta_{sun})$ distributions for year 3, using the amplitudes extracted from the year $1+2+3$ analysis. From fig 4a it is observed that the acrylic background dominates above all the other signals and is comparable to the NC and CC signals at $R \approx 4$ meters. The extracted amplitudes are in poor agreement with those generated and the systematic effects are large since the extracted amplitudes for the ESH_2O , CC and NC are many standard deviations away from the generated values. TABLE VI summarizes the extracted/generated ratios for each of the signals.

5.3 Year $1 + 2 + 3$ Analysis

The situation is much improved over the year 3 only analysis with only the ESH_2O signal being very far off (42%) from the generated value. However while the extracted CC signal is only 7% lower than the generated value, it is still 3.5 standard deviations away. The results are summarized in Table VI.

6 CONCLUSIONS

Lumping all three years of data into one analysis, improves the statistical accuracy of the experiment by a factor of ≈ 2 over the accuracy obtained from analyzing year 3 alone but the use of this technique requires the fundamental assumption of a constant solar signal.

The X100 acrylic radioactivity scenario appears to be untenable. It dominates all the other signals and introduces large systematic errors when the year 3 data is analyzed by itself. The situation improves when the year $1 + 2 + 3$ analysis is used, but systematic effects remain.

TABLE I

x1 Acrylic

Individual Years

Average (RMS)

Reaction	Input	Year 1	Year 2	Year 3
ES H_2O	110	110.2(10.9)	107.9(11.4)	109.5(12.4)
ES D_2O	219	218.8(15.3)	208.1(23.7)	252.5(29.9)
Chg Current	2241	X	2221.3(58.3)	2127.2(109.8)
NC D_2O	149	X	173.5(41.1)	X
Acrylic D_2O	8	X	16.4(24.5)	X
NC Cl	2315	X	X	2377.5(118.1)
Acrylic Cl	234	X	X	252.3(52.6)
NC-CC Corr Coeff	X	X	X	-.717
NC Cl/CC	1.033	X	X	1.118(0.105)

TABLE II

x1 Acrylic

Multiple Years

Average (RMS)

Reaction	Input	Year 1+2	Year 1+3	Year 2+3	Year 1+2+3
ES H_2O	110	109.1(7.9)	110.0(8.2)	108.7(8.4)	109.3(6.8)
ES D_2O	219	215.8(12.7)	226.0(13.7)	226.5(19.1)	221.9(11.9)
CC	2241	2217.6(57.0)	2132.1(110.8)	2195.9(51.6)	2197.8(50.5)
NC D_2O	149	172.5(41.3)	X	175.6(40.3)	176.3(40.4)
Acrylic D_2O	8	16.2(27.5)	X	16.6(25.6)	16.4(27.0)
NC Cl	2315	X	2391.7(118.8)	2342.2(89.0)	2344.1(90.0)
Acrylic Cl	234	X	252.2(55.0)	249.4(52.6)	249.1(52.7)
NC-CC Corr Coeff	X	X	-.731	-.420	-.423
NC Cl/CC	1.033	X	1.122(0.106)	1.067(0.056)	1.067(0.056)

TABLE III

x10 Acrylic

Individual Years

Average (RMS)

Reaction	Input	Year 1	Year 2	Year 3
ES H_2O	110	110.2(10.9)	112.6(11.8)	119.5(14.5)
ES D_2O	219	218.8(15.3)	209.1(24.0)	253.8(32.1)
Chg Current	2241	X	2235.0(58.9)	2015.5(115.6)
NC D_2O	149	X	183.4(42.6)	X
Acrylic D_2O	80	X	58.9(31.6)	X
NC Cl	2315	X	X	2448.3(125.5)
Acrylic Cl	2344	X	X	2388.0(76.7)
NC-CC Corr Coeff	X	X	X	-.722
NC Cl/CC	1.033	X	X	1.215(0.122)

TABLE IV

x10 Acrylic

Multiple Years

Average (RMS)

Reaction	Input	Year 1+2	Year 1+3	Year 2+3	Year 1+2+3
ES H_2O	110	111.3(7.8)	113.7(8.7)	115.5(9.3)	113.4(7.0)
ES D_2O	219	216.1(12.7)	225.4(13.8)	226.9(19.3)	221.9(11.9)
CC	2241	2231.6(57.9)	2020.1(115.5)	2188.2(52.2)	2190.0(51.9)
NC D_2O	149	182.3(42.6)	X	191.5(42.6)	191.9(43.0)
Acrylic D_2O	80	59.5(31.0)	X	59.5(31.6)	60.3(31.6)
NC Cl	2315	X	2462.3(123.4)	2333.6(94.4)	2335.2(94.0)
Acrylic Cl	2344	X	2393.4(76.5)	2378.3(75.6)	2379.9(75.8)
NC-CC Corr Coeff	X	X	-.726	-.410	-.418
NC Cl/CC	1.033	X	1.219(0.122)	1.066(0.058)	1.066(0.058)

TABLE V

x100 Acrylic

Average (RMS)

Reaction	Input	Year 1 + 2 + 3			Year 2	Year 3
	x100 Acr	x1 Acr	x10 Acr	x100 Acr	x100 Acr	x100 Acr
ES H_2O	110	109.3(6.8)	113.4(7.0)	155.8(13.0)	117.6(13.0)	243.7(25.7)
ES D_2O	219	221.9(11.9)	221.9(11.9)	223.0(21.2)	215.5(25.3)	235.7(36.3)
Chg Current	2241	2197.8(50.5)	2190.0(51.9)	2089.6(54.2)	2194.5(60.8)	1542.2(119.2)
NC D_2O	149	176.3(40.4)	191.9(43.0)	215.2(52.2)	184.6(51.5)	X
Acrylic D_2O	811	16.4(27.0)	60.3(31.6)	812.0(54.0)	817.8(54.0)	X
NC Cl	2315	2344.1(90.0)	2335.2(94.0)	2469.5(110.4)	X	2845.9(141.4)
Acrylic Cl	23439	249.1(52.7)	2379.9(75.8)	23420.(173.0)	X	23457.(173.7)
NC-CC Corr Coeff	X	-.423	-.418	-.367	X	-.697
NC Cl/CC	1.033	1.067(0.056)	1.066(0.058)	1.182(0.071)	X	1.845(0.217)

TABLE VI

Extracted/Generated Ratios

X100 Acrylic

Reaction	Year 2	Year 3	/cyrott
ES H_2O	1.07(0.12)	2.21(0.23)	1.42(0.12)
ES D_2O	0.98(0.12)	1.08(0.17)	1.02(.10)
Chg Current	0.98(0.03)	0.69(0.05)	0.93(.02)
NC D_2O	1.24(0.35)	X	1.44(0.35)
Acrylic D_2O	1.01(0.07)	X	1.00(0.07)
NC Cl	X	1.23(0.06)	1.07(0.05)
Acrylic Cl	X	1.00(0.01)	1.00(0.01)

FIGURE CAPTIONS

All figures are for X100 acrylic. The (a) figures are the distributions for each of the reactions and their sum. The (b) figures are the "data" (course histogram), best fit and generating functions (fine histograms) overlayed. The best fit and generating function sometimes overlap so well that they appear as one curve. The best fits are from the amplitudes extracted using the year $1 + 2 + 3$ anlaysis.

Fig 1. Radial R^3 distributions for year 2.

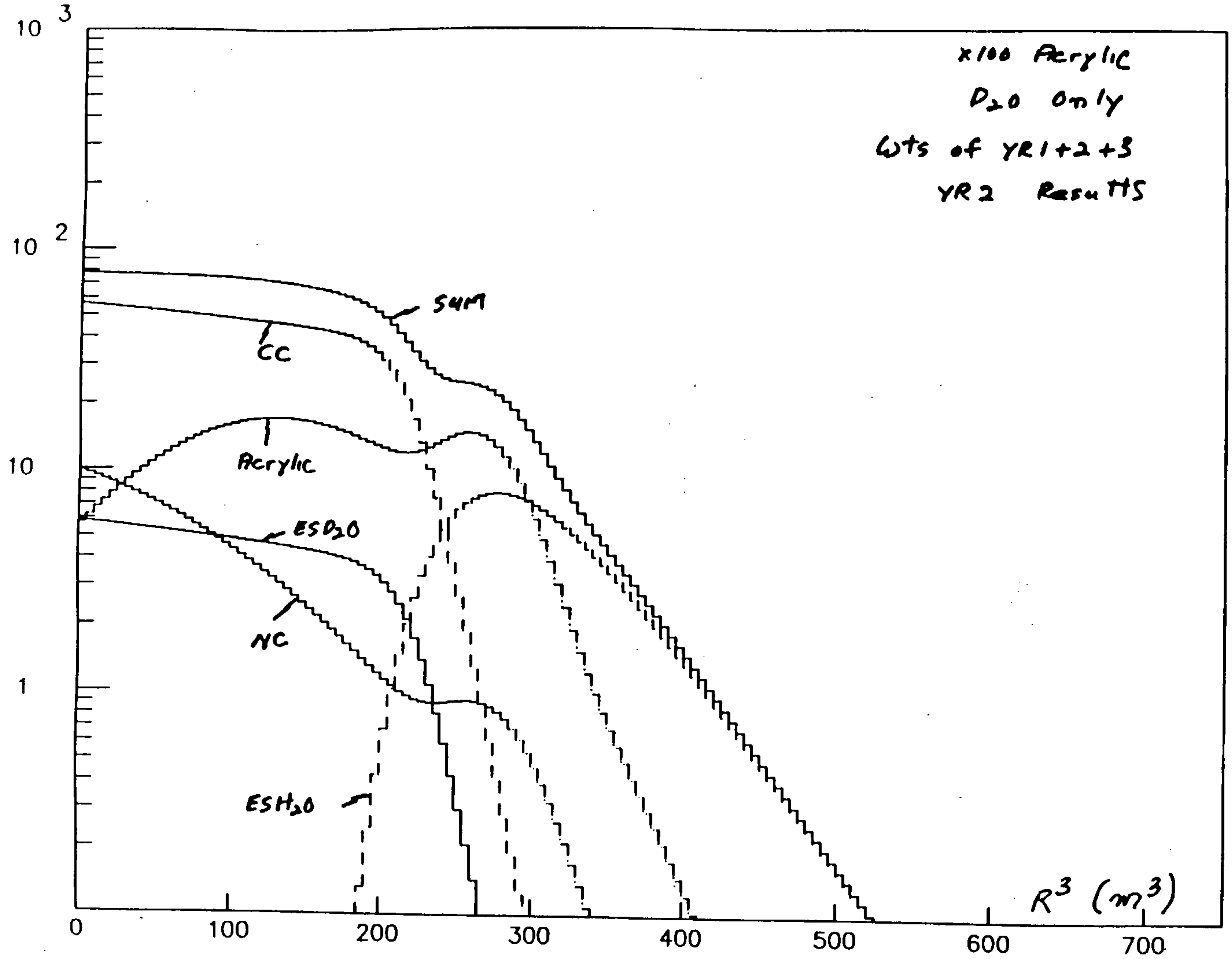
Fig 2. Number of PMT's for year 2.

Fig 3. $\cos(\theta_{sun})$ for year 2.

Fig 4. Radial R^3 distributions for year 3.

Fig 5. Number of PMT's for year 3.

Fig 6. $\cos(\theta_{sun})$ for year 3.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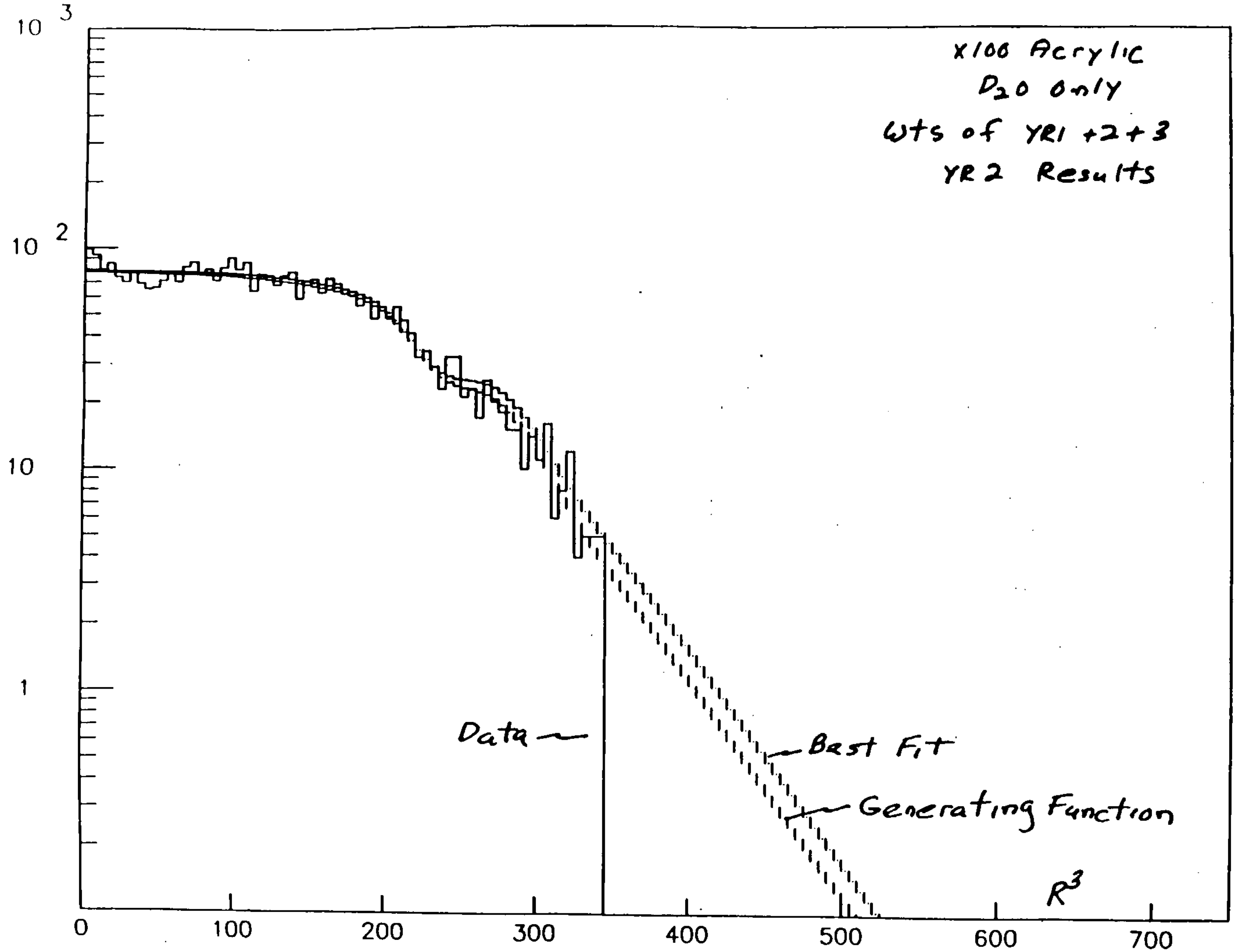
D2O Only
Wts of YR1+2+3
YR2 Results
SUM
CC
Acrylic
ESD2O
NC
ESH2O
10^3
10^2
10
1
0
100
200
300
400
500
600
700
R^3 (m^3)

Fig. 1a


X100 Acrylic
D_2O Only
Wts of YR1 +2+3
YR2 Results
Data
Best Fit
Generating Function
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10^2
10
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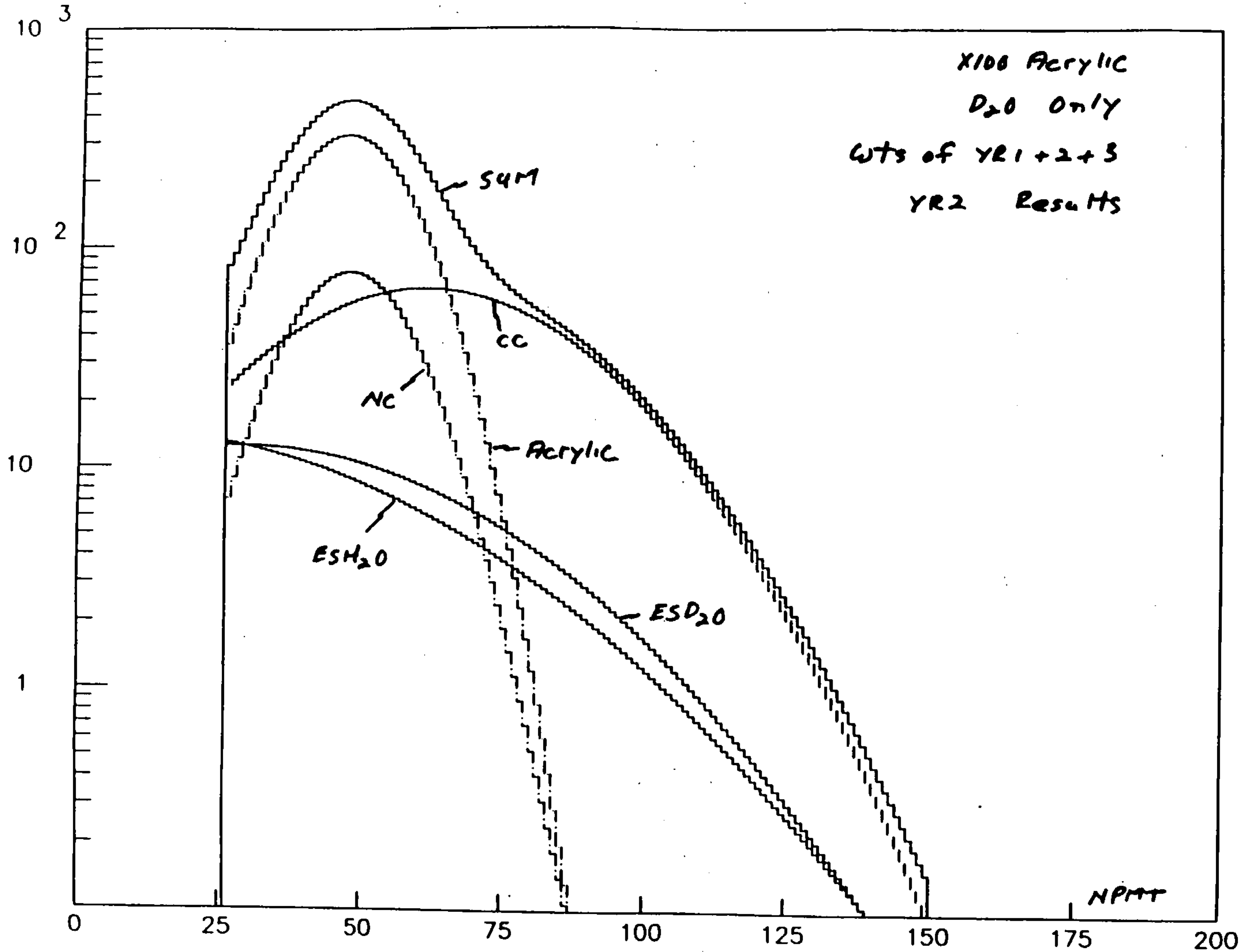
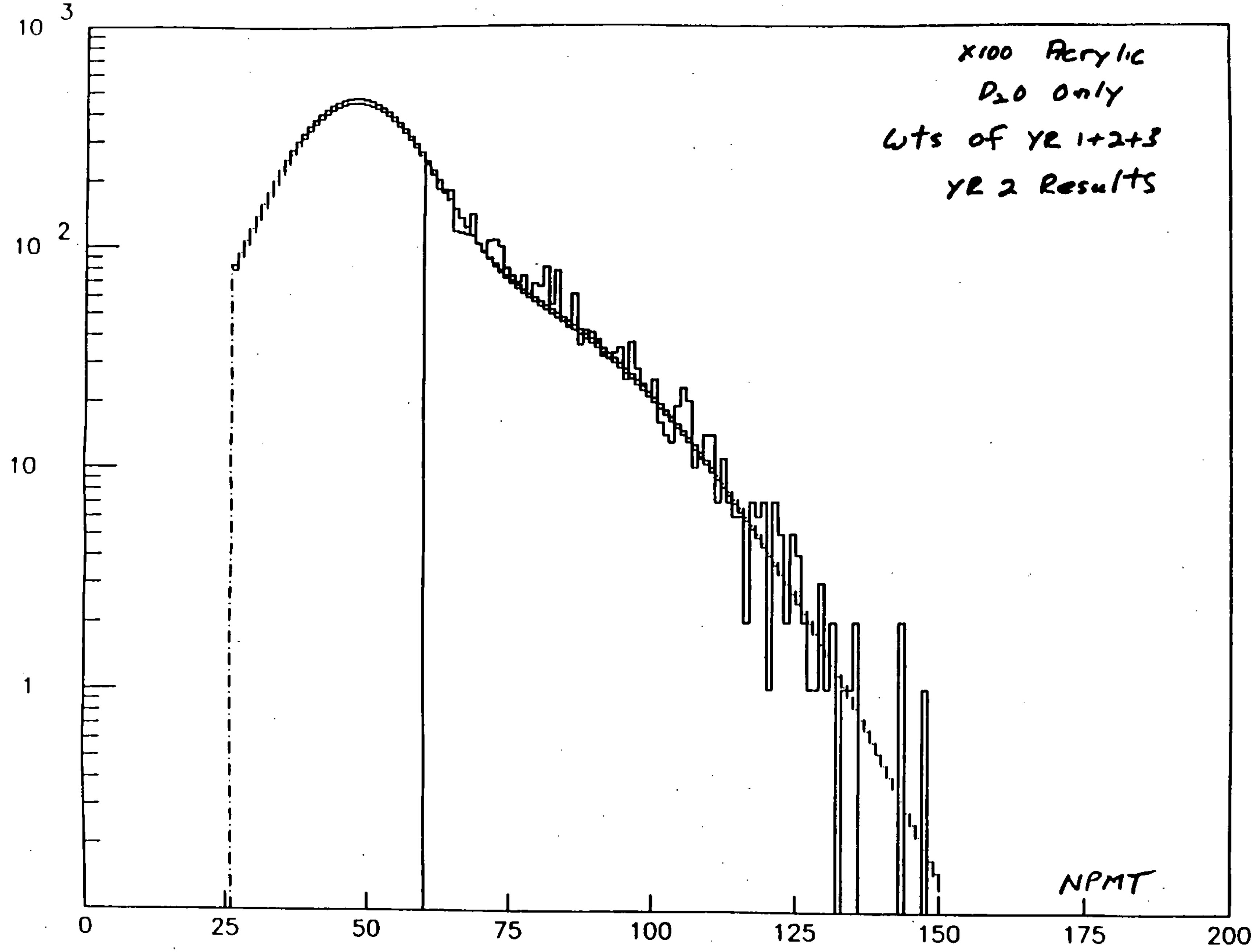
0
100
200
300
400
500
600
700

Fig. 1b



X100 Acrylic
D2O Only
Wts of YR1+2+3
YR2 Results
SUM
CC
NC
Acrylic
ESH2O
ESD2O
NPMT
10 3
10 2
10
1
0
25
50
75
100
125
150
175
200

Fig. 2a



Fig

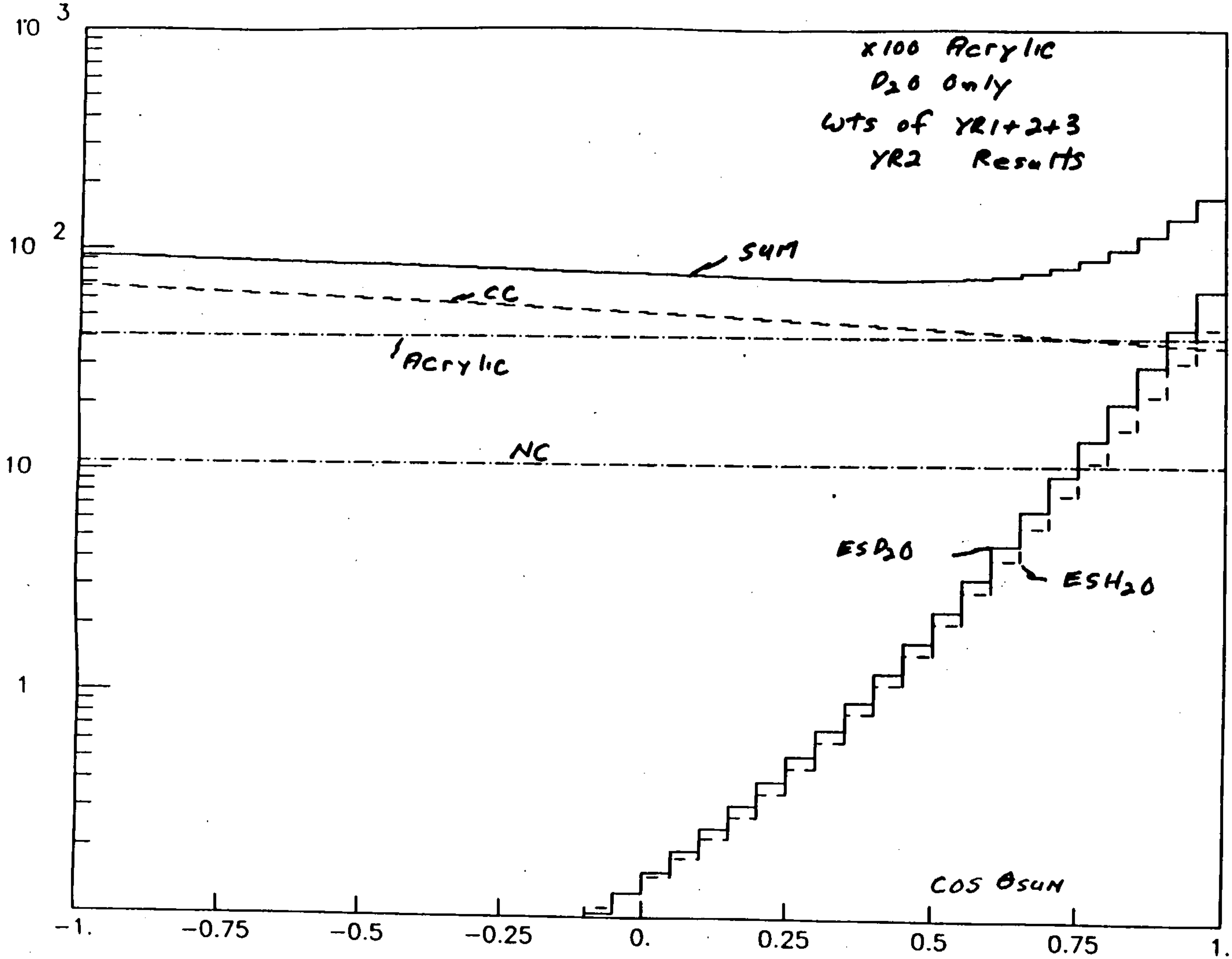
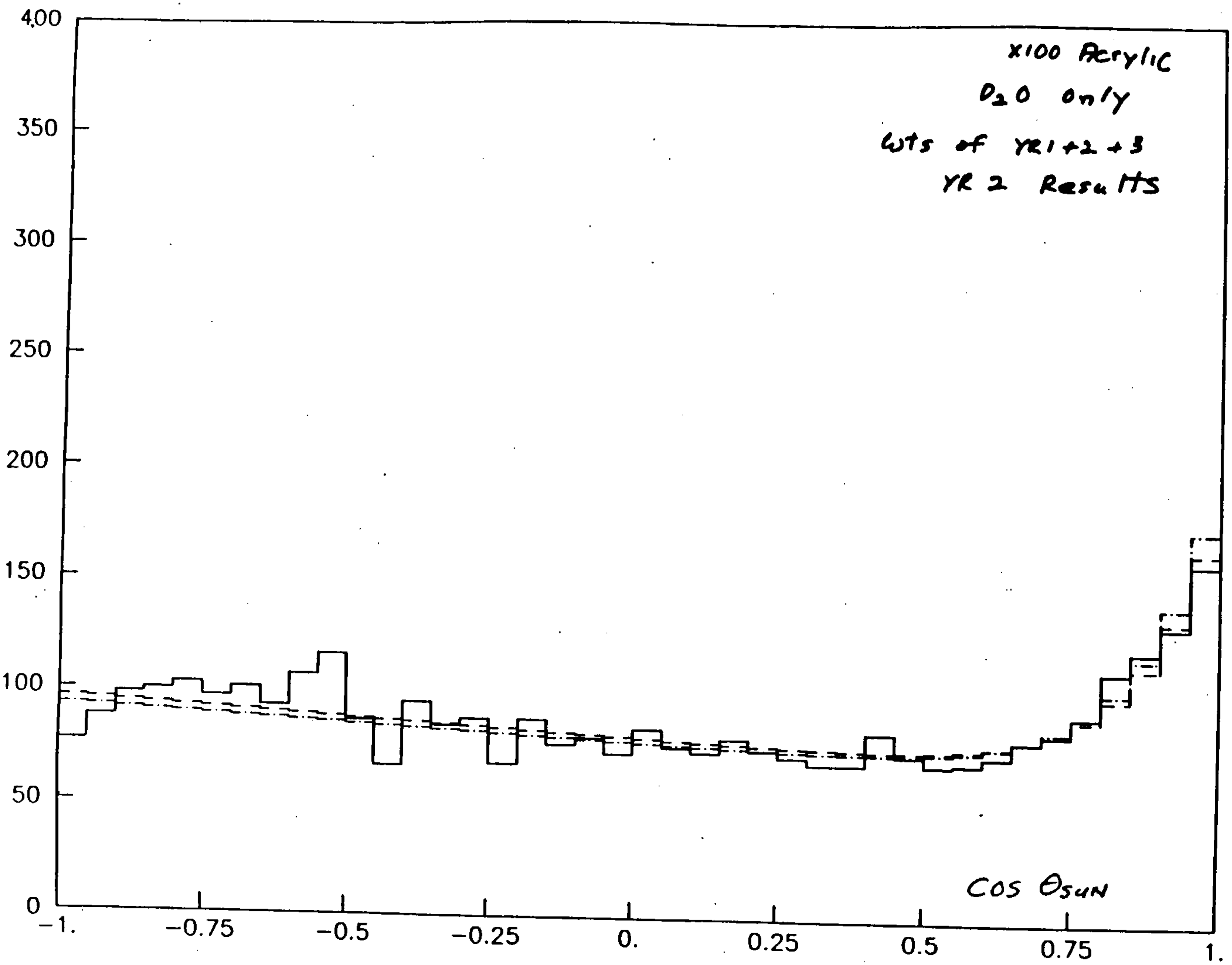

x100 Acrylic
D2O Only
Wts of YR1+2+3
YR2 Results
SUM
CC
Acrylic
NC
ESD2O
ESH2O
COS θSUN
10 3
10 2
10
1
-1.
-0.75
-0.5
-0.25
0.
0.25
0.5
0.75
1.

Fig. 3a



X100 Acrylic
D2O only
Wts of YR1+2+3
YR 2 Results
400
350
300
250
200
150
100
50
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-0.5
-0.25
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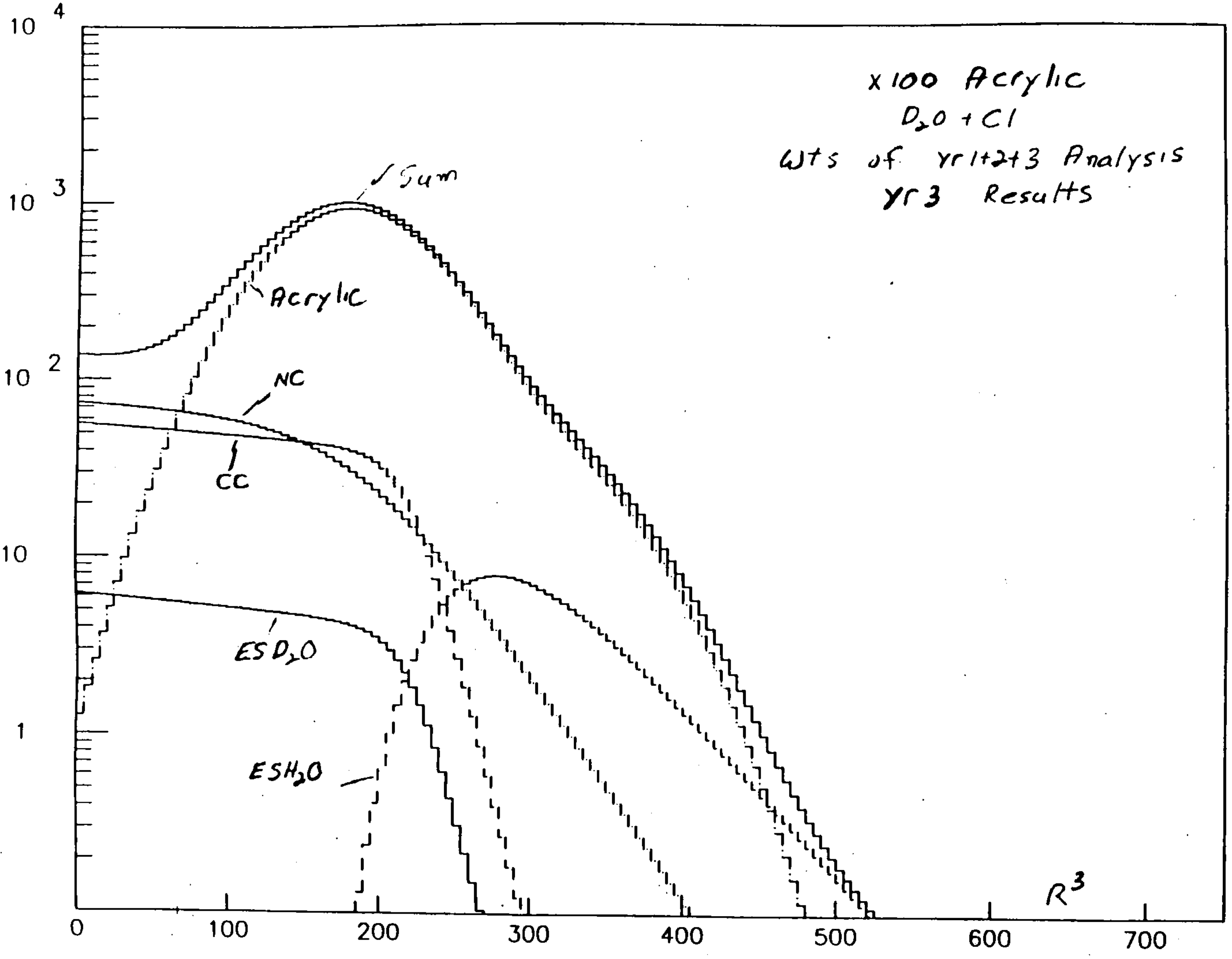
0.25
0.5
0.75
1.
Cos θsun

Fig.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D_2O + Cl
Wts of yr1+2+3 Analysis
yr3 Results
Sum
Acrylic
NC
CC
ESD_2O
ESH_2O
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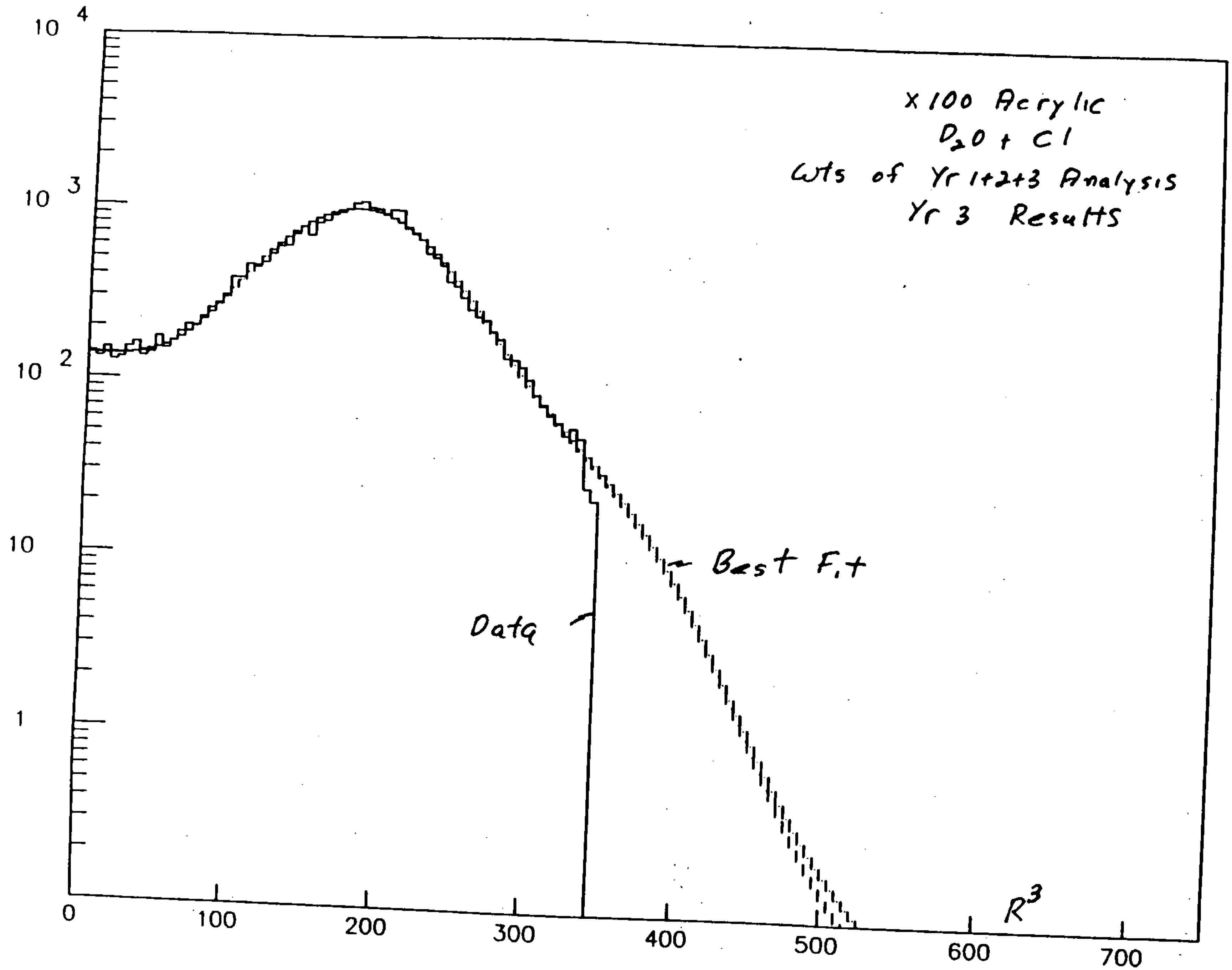
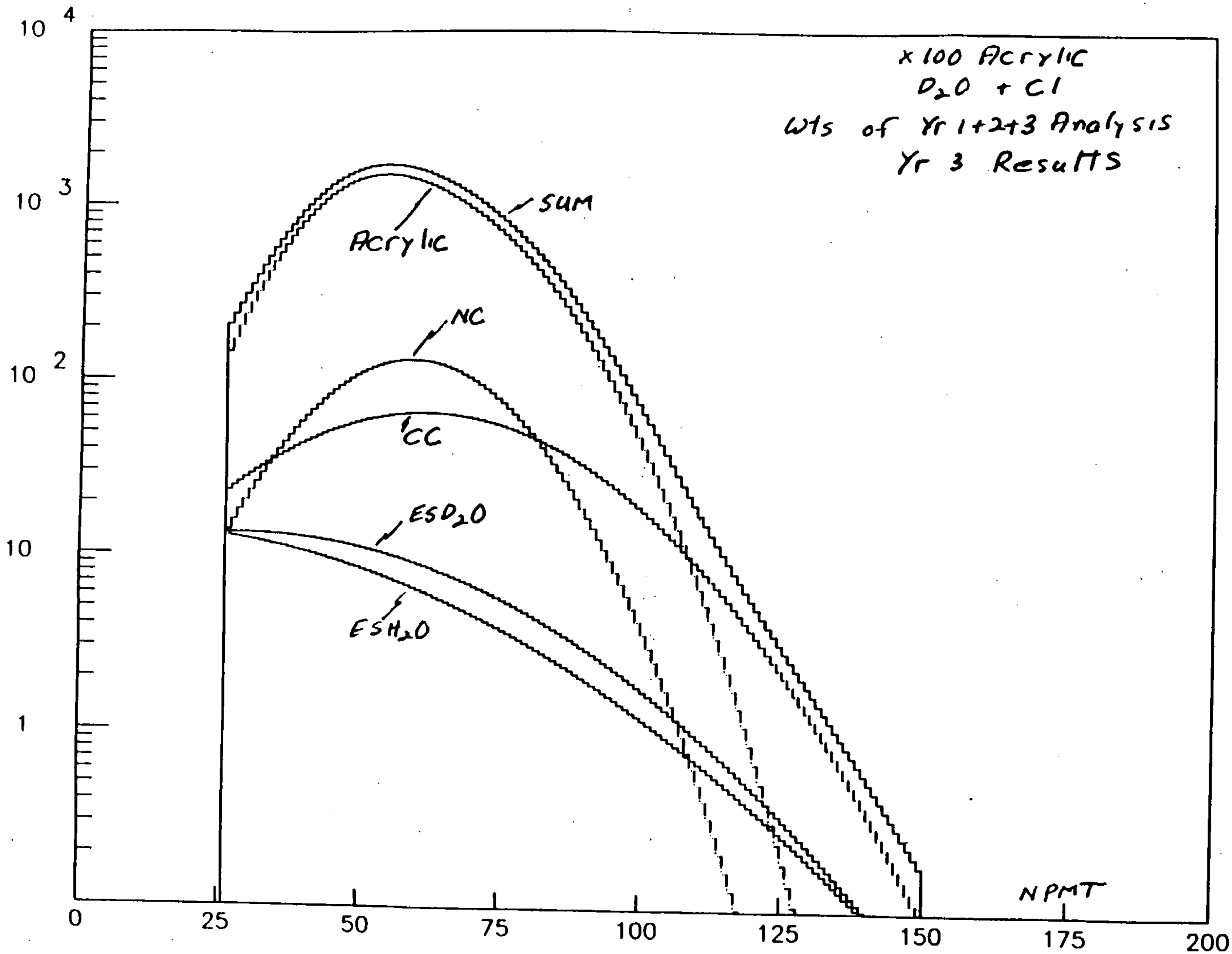
10^3
10^2
10
1
0
100
200
300
400
500
600
700
R^3

Fig. 4a



x 100 Acrylic
D_2O + Cl
Wts of Yr 1+2+3 Analysis
Yr 3 Results
Best Fit
Data
10^4
10^3
10^2
10
1
0
100
200
300
400
500
600
700
R^3

Fig.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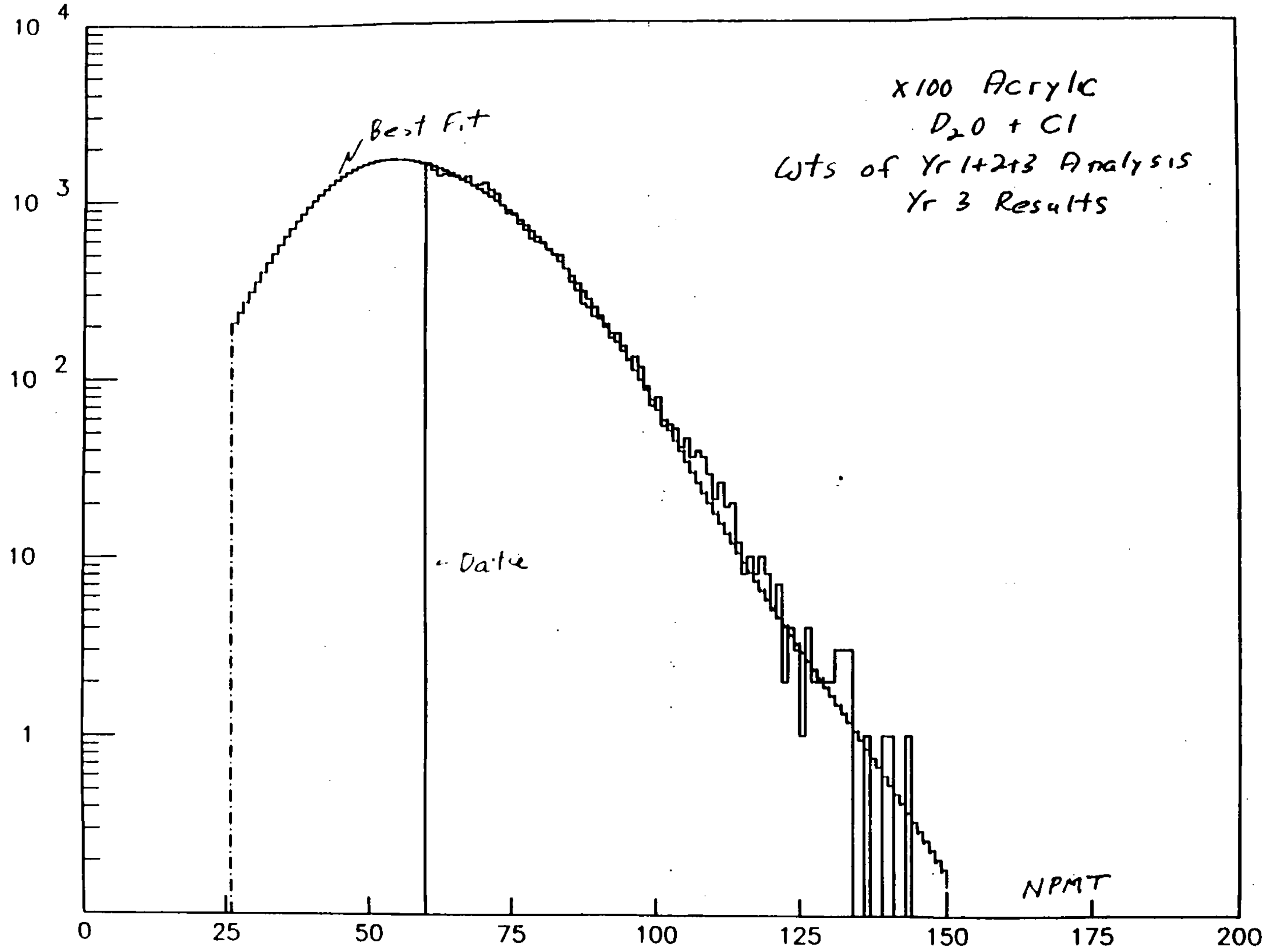
D_2O + Cl
Wts of Yr 1+2+3 Analysis
Yr 3 Results
SUM
Acrylic
NC
CC
ESD_2O
ESH_2O
NPMT
10^4
10^3
10^2
10
1
0
25
50
75
100
125
150
175
200

Fig. 5a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D_2O + Cl
Wts of Yr 1+2+3 Analysis
Yr 3 Results
Best Fit
← Data
NPMT
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10^3
10^2
10
1
0
25
50
75
100
125
150
175
200

Fig. 5b

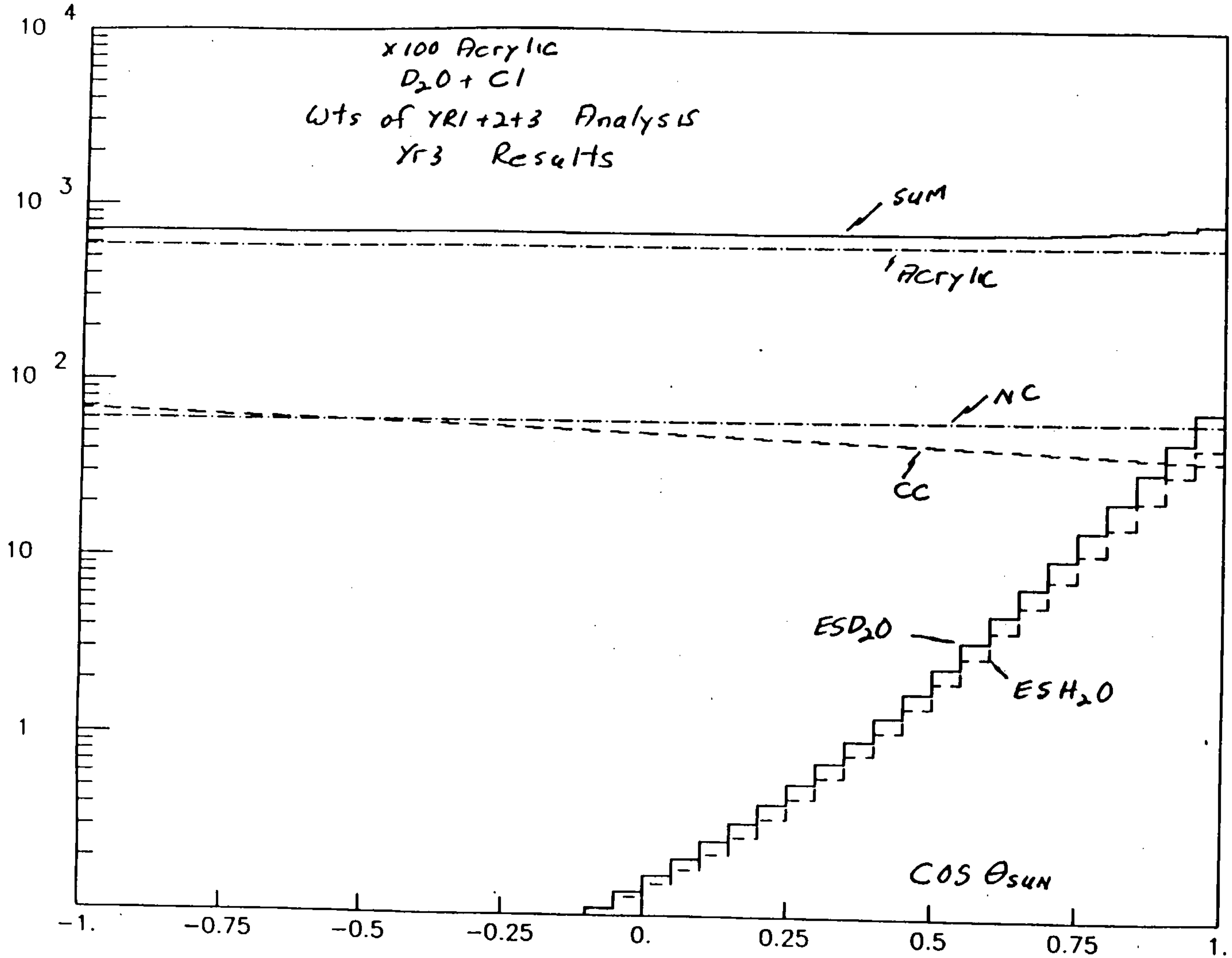
x100 Acrylic
D_2O + Cl
Wts of YR1+2+3 Analysis
Yr3 Results
SUM
Acrylic
NC
CC
ESD_2O
ESH_2O
COS θsun
10^4
10^3
10^2
10
1
-1.
-0.75
-0.5
-0.25
0.
0.25
0.5
0.75
1.

Fig. 6a

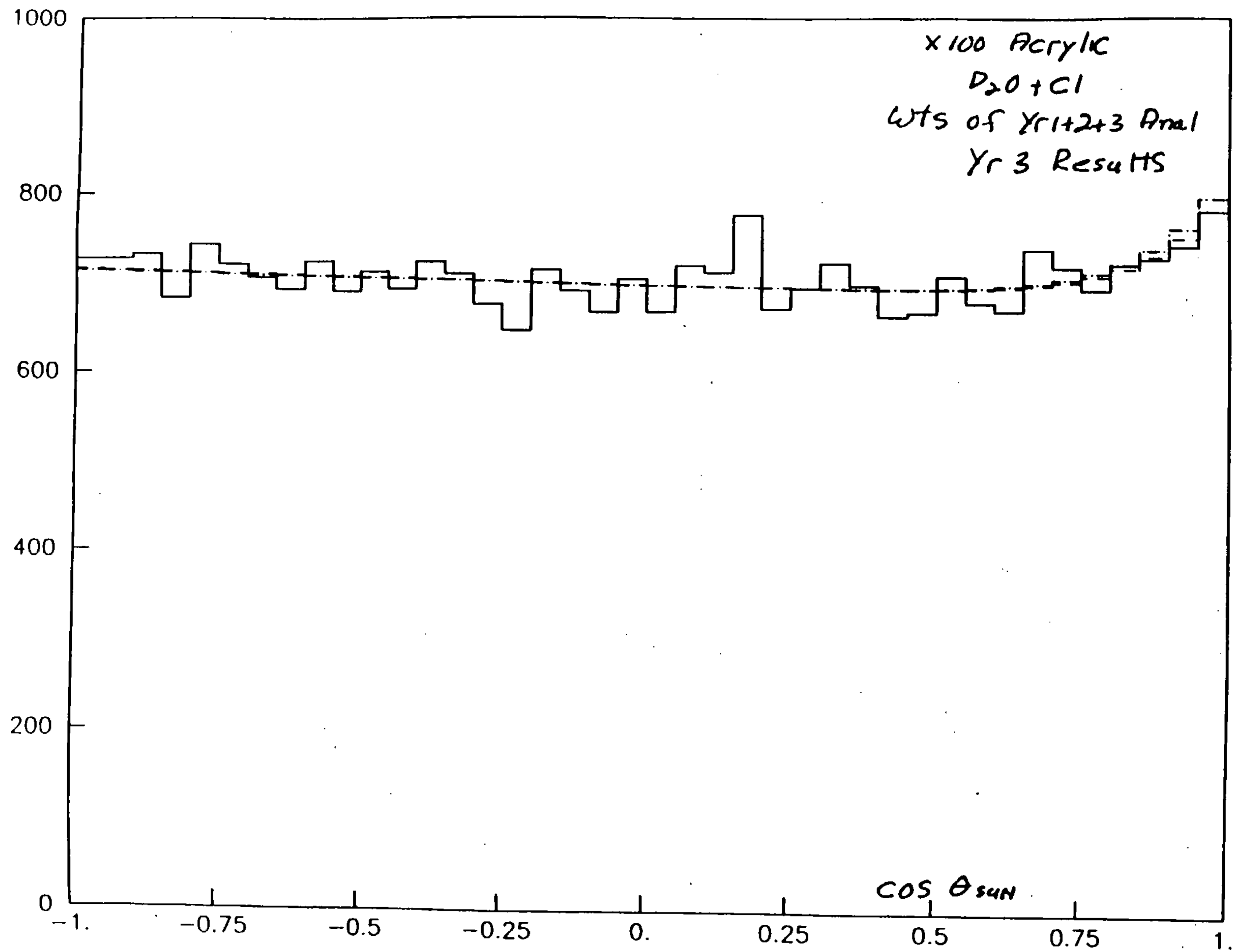
x 100 Acrylic
D2O + Cl
Wts of Yr 1+2+3 Anal
Yr 3 Results
1000
800
600
400
200
0
−1.
−0.75
−0.5
−0.25
0.
0.25
0.5
0.75
1.
cos θsun

Fig.